



# Fermi National Accelerator Laboratory

FERMILAB-Conf-85/15  
0102.000

## MEASURING STRUCTURE FUNCTIONS AT SSC ENERGIES\*

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January 1985

\*Submitted to the Proceedings of Snowmass '84, Snowmass, Colorado, June 23 - July 13, 1984.



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### 1. Summary

This is a summary of the working group on structure functions. The members of the working group were

J. C. Collins, Illinois Institute of Technology;  
S. Loken, University of California, Berkeley;  
J. G. Morfín, Fermilab;  
J. F. Owens, Florida State University;  
Wu-Ki Tung, Illinois Institute of Technology; and  
G. S. Tzanakos, Columbia University

Topics discussed include measuring  $A$ , tests of QCD using hard scattering processes, and measuring parton distributions. In each case, any opportunities and advantages afforded by the unique features of the SSC are emphasized.

The working group on structure functions was charged with investigating two specific questions: (i) How well are the various parton distributions known in the kinematic region relevant to calculations for the SSC? (ii) What new information can be learned about parton distributions at the SSC? Especially for this working group, the advantages of having a fixed-target facility at the SSC for the measurement of the parton distributions with multi-TeV leptons, were to be examined.

These two questions immediately suggest a whole host of secondary topics, many of which overlap with those of other working groups. By way of background information, it should be recalled that virtually all of the tests of QCD to date have been based on the use of various hard scattering processes. The fact that QCD possesses the feature of asymptotic freedom allows one to employ perturbation theory for the calculation of such processes. However, for reactions with hadrons occurring explicitly in the initial state, it is necessary to augment the formal theoretical calculations with a prescription which gives the momenta of the incoming quanta. The parton model and its associated parton distribution functions provide such a framework. For the case of nucleons, the parton distribution functions (pdf's) for quarks have been measured in deep inelastic lepton nucleon scattering, (although there is not yet complete agreement among experiments, particularly in the low- $x$  regions.) Some information on the gluon distribution in nucleons has also been obtained. For unstable particles such as pions, useful information has been obtained, for example, from high mass dilepton measurements.

Typically, pdf measurements have covered the kinematic range  $0.05 < x < 0.7$  and  $Q^2 < 200$  (GeV/c)<sup>2</sup>. This matches well with the range of parton momentum fractions and momentum transfers which are required for various hard scattering calculations such as high- $p_T$  scattering, heavy resonance production, etc. With the advent of the SSC, however, one is dealing with a whole new kinematic region. For example, the  $Q^2$  range will easily extend up to  $10^8$  (GeV/c)<sup>2</sup> or more and the required  $x$

values may range down to  $10^{-4}$  or less. Especially at such small  $x$  values, the relevance of the parton picture must be questioned. Furthermore, there is the question of the scaling violations which are calculated using, for example, the Altarelli-Parisi evolution equations. These questions were studied by a number of people in various working groups and detailed reports on their findings are contained in these proceedings. Two points are worth emphasizing, however. The question of the relevance of the parton model approach at low  $x$  and high  $Q^2$  was the main point of focus for the small  $x$  working group. Their conclusion, based in large part on the work of Ref. 1, is that one can indeed safely use the parton model approach even down to the region  $x \approx 10^{-4} - 10^{-3}$ , i.e., the region relevant for calculating  $W$  production processes. With this conclusion in hand, one then has to face the problem of obtaining accurate solutions of the evolution equations for small  $x$  and high  $Q^2$  values. Wu-Ki Tung has studied these problems in detail and his report is contained in these proceedings.

The remainder of this report is organized as follows. In Section 2 some recent evidence for a universal pattern of scaling violations for parton distributions in hard scattering processes is briefly reviewed. In so doing, further evidence for the validity of the hard scattering approach will be discussed. In Section 3 the problem of measuring  $A$  is discussed while Section 4 is concerned with various sources of uncertainty in parton distributions. Finally, Section 5 contains a summary of our conclusions.

### 2. Scaling Violations in Hard Scattering Processes

Until recently the logarithmically varying  $Q^2$  dependent scaling violations predicted by QCD had been observed only in deep inelastic lepton nucleon scattering in a  $Q^2$  range extending up to about 200 (GeV/c)<sup>2</sup>. Recently, however, new data on high- $p_T$  jet production from the CERN SppS collider have become available.<sup>2,3</sup> Both the UA-1 and UA-2 collaborations have reported analyses of di-jet production in which they obtained measurements of effective structure functions which are given, approximately, by linear combinations of the usual quark and gluon distributions. Basically, if one measures both of the opposite side high- $p_T$  jets, then the kinematics of the underlying parton-parton scattering can be reconstructed on an event-by-event basis. One can then measure the angular distribution of the parton-parton scattering in the parton-parton center-of-momentum frame. In principle, the cross section is given by a sum over all possible parton-parton subprocesses

$$d\sigma/dx_a dx_b d\cos\theta = \sum_{ab} G_{A/A}(x_a, Q^2) G_{B/B}(x_b, Q^2) d\sigma/d\cos\theta(ab \rightarrow jj). \quad (1)$$

However, it was noticed by the authors of Ref. 4 that the angular distributions of the dominant subprocesses were very similar in shape owing to the

presence of a t-channel pole in each. Thus, to a first approximation, the angular distribution can be factored out of the sum. Next, they observed that the remaining summation can be replaced by the product of two effective distribution functions defined as

$$F(x, Q^2) = x \{ G(x, Q^2) + 4/9 [Q(x, Q^2) + \bar{Q}(x, Q^2)] \}. \quad (2)$$

The factor of 4/9 is from the ratio of color factors and reproduces the correct result in the limit of  $\cos\theta \rightarrow \pm 1$ , which is the region which dominates the cross section.

In the analysis of Ref. 2 a multiplicative "K factor" was included in Eq. (1) in order to approximately take into account possible higher order effects. A value of  $K=2$  was used by the UA-1 group for the determination of  $F(x, Q^2)$  whereas  $K=1$  was used in the UA-2 analysis.<sup>3</sup> There is some theoretical motivation for such a factor. Antoniou et al.<sup>5</sup> have calculated the  $\alpha_s^3$  corrections to the two-body subprocesses in the soft gluon approximation. Their results give a value of  $K \approx 1.6$  for the dominant  $gg$  and  $gq$  subprocesses. It has been shown that in some instances these terms yield a reasonably good approximation to the full corrections. However, there is no way to know this in advance and a detailed higher order calculation is required in order to check each case as is discussed, for example, in Ref. 6.

In Fig. 1, the experimental results from both collaborations for  $F(x, Q^2)$  are shown along with some predictions based upon the distributions from Ref. 7. The UA-1 results have been multiplied by  $2^{1/2}$  in order to remove the effect of the K factor used in their analysis. The dashed curve corresponds to the input distributions<sup>7</sup> of Set 1 ( $\Lambda = 200$  MeV/c) evaluated at  $Q^2=4$  (GeV/c)<sup>2</sup> whereas the lower solid curve was obtained using  $Q^2=2000$  (GeV/c)<sup>2</sup>. The result of including an overall K factor of 1.6 is shown by the upper solid curve. The effect of the calculated scaling violations is quite dramatic and the  $Q^2$  dependence is clearly required in order to describe the shape of the data. This comparison confirms, at the leading logarithm level, the relevance of the concept of universal parton distributions with characteristic  $Q^2$  dependences which may be calculated independently of the hard scattering process. In addition, the average angular distribution obtained for the parton-parton subprocesses agrees well with the expectations based on QCD as is shown in Fig. 2. Finally, it is well known that the leading logarithm QCD predictions are in good agreement with the latest single inclusive jet production data as can be seen in Ref. 8. The overall picture based on these new results serves to enhance the confidence that one can have in the QCD-parton model approach to calculating cross sections for hard scattering processes.

It is encouraging to note that the pattern of scaling violations appears to be correct after extrapolating over approximately one order of magnitude in  $Q^2$ . Calculations of hard scattering processes at the SSC will call for an extrapolation over an additional three orders of magnitude. However, due to the logarithmic nature of the scaling violations, the additional changes in  $F(x, Q^2)$  are less dramatic than at the lower  $Q^2$  values. For comparison, the dotted curve in Fig. 1 shows the prediction for  $F(x, Q^2)$  at  $Q^2 = 1$  (TeV/c)<sup>2</sup>. In this range of  $Q^2$  the most dramatic effects are at  $x$  values too small to be seen clearly on the scale used in the figure. It will, furthermore, be a

severe challenge for experimenters to accurately measure the ultra low  $x$  structure function behavior.

The method of analysis discussed in this section is capable of providing useful information on the parton distributions (primarily that of the gluons) at small values of  $x$ . The primary limitation at this time is the lack of detailed higher order calculations for the relevant subprocesses. Nevertheless, one can anticipate obtaining qualitative information concerning the evolution of the parton distributions using this technique.

### 3. Measurements of $\Lambda$

The most precise information available to date for the parton distributions in nucleons has come from the study of deep inelastic lepton-nucleon scattering. The available data show the pattern of scaling violation expected on the basis of QCD. In addition to providing measurements of the pdf's themselves, these data have also made possible estimates of the QCD scale parameter  $\Lambda$ , both in leading and next-to-leading order. The results indicate that  $\Lambda_{LO} \approx 200-300$  MeV, but with a rather large uncertainty which, in most instances, is dominated by systematic errors. The determination of  $\Lambda$  in deep inelastic scattering is discussed in detail<sup>9</sup> while the determination of  $\alpha_s$  in general has been recently reviewed.<sup>10</sup>

At the energies presently available, measurements of  $F_2$  alone, as obtained from electron or muon nucleon deep inelastic scattering, are insufficient for a precise determination of  $\Lambda$ . This is due, in part, to the correlation that exists between the gluon distribution and the fitted value of  $\Lambda$ . A harder gluon distribution results in a larger value of  $\Lambda$  and vice versa. Within a rather broad range of  $\Lambda$  values the quality of the various fits is unchanged. There are several ways out of this dilemma, the most straightforward of which is to have high statistics data on the nonsinglet structure function  $xF_3$ , as measured in neutrino nucleon scattering. Present day analyses are still statistics limited. Although this situation is expected to improve as additional data are accumulated, the currently approved neutrino programs in both Europe and the U.S. will probably not yield sufficient statistics to allow an accurate determination of  $\Lambda$  with  $xF_3$ .

Another way of reducing the errors on  $\Lambda$  is to utilize a larger range in  $Q^2$ . A fixed target program at the SSC with lepton beams would greatly extend the range of  $Q^2$  over that obtainable at other fixed-target facilities. It would, in fact, be comparable to that expected at HERA. The SSC fixed-target facility would have certain advantages over an ep facility such as HERA among which are: 1) the number of available beam types would be larger therefore enabling a greater variety of measurements to be made; 2) a fixed-target facility could immediately employ isoscalar targets which facilitates the extraction of the structure functions; HERA would have to be able to accelerate deuterons to use this isoscalar advantage; and 3) the loss of particles down the beam pipe will result in large smearing corrections and large systematic errors for the HERA detectors. A detailed study of the relative merits of such a fixed-target option has been performed and included in the proceedings of the Texas Fixed Target Workshop. Current analyses are limited to  $Q^2 \leq 200$  (GeV/c)<sup>2</sup> whereas with the fixed-target option the upper limit of the useful  $Q^2$  range would be extended to approximately 15,000

(GeV/c)<sup>2</sup>. Being able to reach higher values of  $Q^2$  also allows the experimenters to increase the lower  $Q^2$  bound and thus to further reduce the importance of higher twist effects. In Ref. 7 two sets of parton distribution parametrizations were given, corresponding to  $A=200$  and  $400$  MeV/c, and referred to as Set 1 and Set 2, respectively. Figure 3 shows predictions for  $F_2$  at several values of  $x$  for these two sets of distributions. At the highest  $Q^2$  value shown there is approximately a 10% difference between the two curves. This shows the level of combined systematic and statistical errors that must be reached in order to reduce the level of uncertainty in  $A$ . A detailed discussion of event rates and the systematic errors expected from a typical fixed-target detector is described in a separate report<sup>15</sup> from the fixed-target working group. In brief, it was found that event rates, which will depend on the type of extraction scheme employed and the type of beam used, will range from 15000 to over 220000 neutrino events per week for a 10 ton neutrino detector and from 1.5 to 15 times as many muon events (in a 10m D target) as HERA will produce per week with  $L = 5 \times 10^{31}$ . As far as systematic errors are concerned, taking a typical muon experiment with calorimetry, over the entire  $x_B$ - $y$  plane the shift in  $x$  and  $Q^2$  should be less than  $5 \times 10^{-3}$  and the resolution in  $x$  and  $Q^2$  will be of the order of 2%. This is superior to the capabilities of the HERA detectors presently envisioned.

Determinations of  $A$  are not limited to deep inelastic scattering processes, of course. However, in order to determine  $A$  in a meaningful way using a number of different processes, the next-to-leading order calculations for them must be done. To date, complete calculations have been done for dilepton production,<sup>12</sup> high- $p_T$  direct photon production,<sup>6</sup> and gauge boson production<sup>13</sup> in addition to deep inelastic scattering.<sup>12</sup> Thus, even without the fixed-target option there will still be possible methods for determining  $A$ .

In deep inelastic scattering it is the derivative of the structure function with respect to the logarithm of  $Q^2$  which is proportional to  $\alpha_s$  and which, therefore, provides an estimate of  $A$ . The normalization of the structure functions at a particular value of  $x$  is primarily determined by the fitted input parton distributions. The situation is somewhat different for high- $p_T$  processes involving one or more hadronic jets. There the cross section is proportional to one or more powers of  $\alpha_s$  and, therefore, the normalization of the cross section itself provides a constraint on  $A$ . In addition, the normalization also depends on the relevant parton distributions which, in turn, provides another indirect constraint on  $A$  via the evolution equations. A comprehensive analysis would, therefore, consist of simultaneously fitting data from deep inelastic scattering and other hard scattering processes. The free parameters would include  $A$  and those necessary to describe the input parton distributions.

Another point to remember is one related to the  $A$ -gluon correlation mentioned previously for deep inelastic scattering. As long as one uses a consistent set of parton distributions with the appropriate  $A$  value for making predictions for hard scattering processes the apparent sensitivity to choices of the gluon distribution will be reduced. This is shown explicitly<sup>7</sup> where nearly identical predictions for direct photon production were obtained using the two different sets of parton distributions. Similar results were also found for

high- $p_T$  hadron production. This further emphasizes the need for fitting a variety of different observables each of which is sensitive to different combinations of parton distributions and which are subject to different systematic errors.

#### 4. Uncertainties in Parton Distributions

When making predictions for hard scattering processes in new kinematic regions it is important to have at least a qualitative estimate regarding the uncertainties in the parton distribution functions used. Assuming that the problems of a strictly numerical nature are under control, there can still be differences resulting from variations in  $A$ , the initial parton distributions, the treatment of heavy flavors, etc. Several of these points have been discussed in detail.<sup>8</sup>

First, consider the evolution of the parton distributions. It is important to remember that the distributions at large  $x$  and small  $Q^2$  feed down to the distributions at small  $x$  and large  $Q^2$  as a result of the structure of the evolution equations. For example, radiation of gluons from the valence quarks causes the gluon distribution to change. Similarly, the gluons feed the quark sea via  $q\bar{q}$  pair creation. Of all of the distributions, those of the valence quarks are the best measured. Therefore, differences due to variations in the initial gluon and sea distributions tend to be reduced at high  $Q^2$  since a large part of the evolution comes from the better known valence distributions. Another point to remember is that at very small  $x$  values none of the distributions are well measured. Thus, one might expect that this would lead to additional uncertainties in the small  $x$  high  $Q^2$  parton distribution extrapolations. However, the same feed down effect mentioned above helps here, as well. The dominant sources of low  $x$  high  $Q^2$  partons are at larger  $x$  and lower  $Q^2$  values. The basic conclusion, then, is that the structure of the evolution equations tends to reduce the effects of the uncertainties in the initial distributions when one goes to very high values of  $Q^2$ .

Another source of uncertainty in the evolved parton distributions is related to the treatment of heavy flavors. The  $Q^2$  range available at the SSC necessitates the inclusion of  $b$  and  $t$  quark distributions in the calculation of many hard scattering processes. The usual technique for estimating heavy quark distributions is to assume that they are zero below some threshold value of  $Q^2$ . As the threshold is crossed, the number of flavors is increased by one and the corresponding heavy quark distribution is generated by the evolution equations. Usually this procedure is carried out using the massless splitting functions so that the estimates are reliable only when one is far above the respective threshold. In principle, one can (and should) include the effects of the quark masses in the evolution equations in a consistent manner. Some efforts in this direction are currently underway.

#### 5. Summary and Conclusions

The following is a list of the major points relevant to parton distributions which were discussed during the summer study. Where appropriate, references to detailed reports contained in these proceedings are given.

a) It appears likely that the parton model prescription will still be applicable in the small  $x$

and high  $Q^2$  regions relevant for hard scattering calculations at the SSC. This topic is discussed in detail in the report of the small  $x$  working group.

b) The numerical problems encountered for  $x < 10^{-3}$  and  $Q^2 > 10^5$  (GeV/c)<sup>2</sup> can be controlled in a relatively straightforward fashion. This topic is discussed in detail by Wu-Ki Tung in his report.

c) Di-jet analyses following<sup>4</sup> may be used to provide qualitative information on the evolution of parton distributions in the high  $Q^2$  region available at the SSC. The primary restriction on the use of this technique is the lack of the relevant next-to-leading order calculations. However, there are other hard scattering processes for which the calculations have been done and which should be observable at the SSC. A prime example is direct photon production.

d) For measurements of  $\Lambda$  one can get some information from the various hard scattering processes alluded to above. However, the  $\Lambda$ -gluon correlation implies that a joint analysis of a variety of processes will be required to significantly reduce the error on  $\Lambda$ . In this regard, the fixed-target option is crucial for obtaining important information from deep inelastic scattering.

e) Current parton distributions differ most in the choice of the gluon distribution and the corresponding fitted value of  $\Lambda$ , reflecting our current lack of detailed information on the gluon distribution. This situation will change for the better with improved measurements of the non-singlet structure function  $xF_3$ . However, only moderate improvement can be expected in the foreseeable future from the currently approved neutrino program. A high statistics experiment in the expanded  $x$ - $Q^2$  range available at the SSC fixed-target facility would be invaluable in resolving

this situation. For now, the best that one can do is to give representative sets of distributions which reflect these uncertainties.

f) There is room for improvement in the calculation of the heavy quark distributions, most notably by the inclusion of mass effects.

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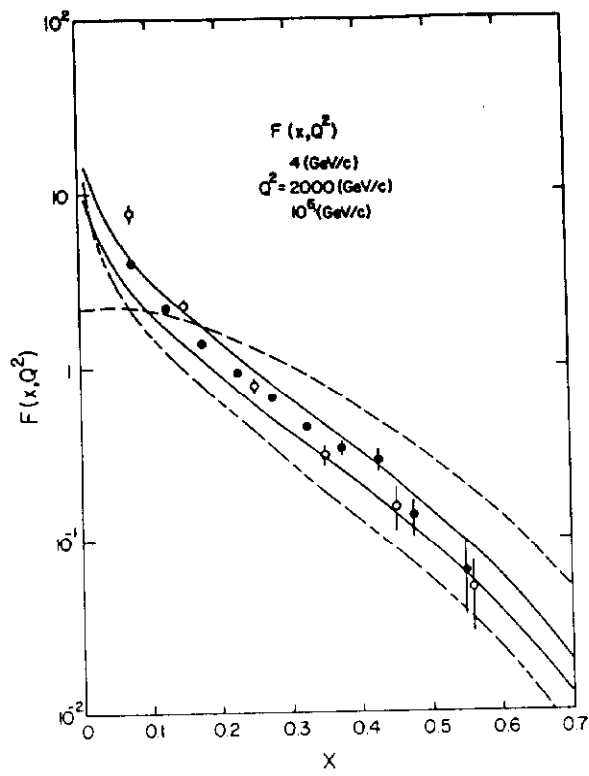


Fig. 1. Results for the effective structure function  $F(x, Q^2)$  as defined in Eq.(2). The data are from Ref. 2 (open circles) and Ref. 3 (solid circles). The curves are predictions based on the Set 1 distributions.<sup>7</sup> The dashed curve has  $Q^2 = 4 \text{ (GeV/c)}^2$  while the lower solid curve has  $Q^2 = 2000 \text{ (GeV/c)}^2$ . The upper solid curve has been scaled upwards by a "K factor" of 1.6. The dotted curve is the prediction for  $Q^2 = 1 \text{ (TeV/c)}^2$ .

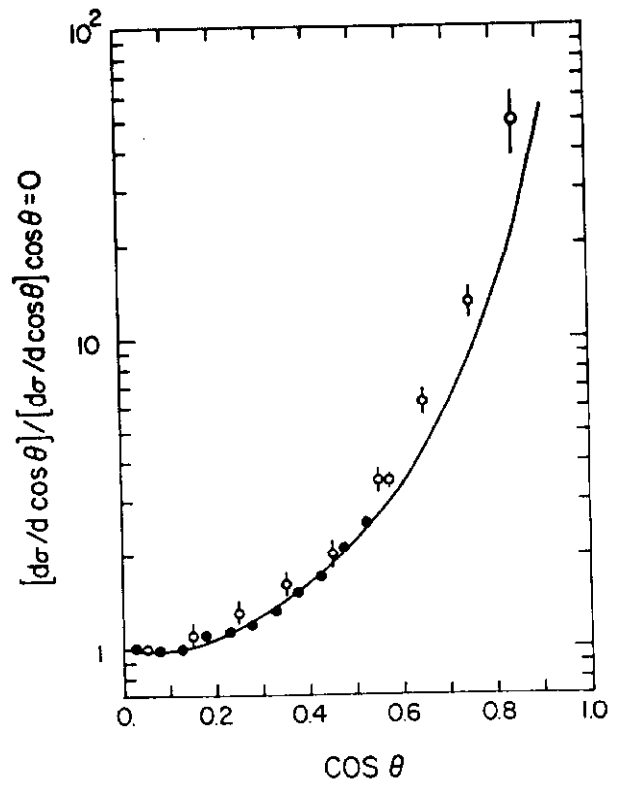


Fig. 2. The effective parton-parton angular distribution in the parton-parton center-of-momentum frame. The data are from Ref. 2 (open circles) and Ref. 3 (solid circles). The curve is the prediction of QCD based on two-body scattering subprocesses.

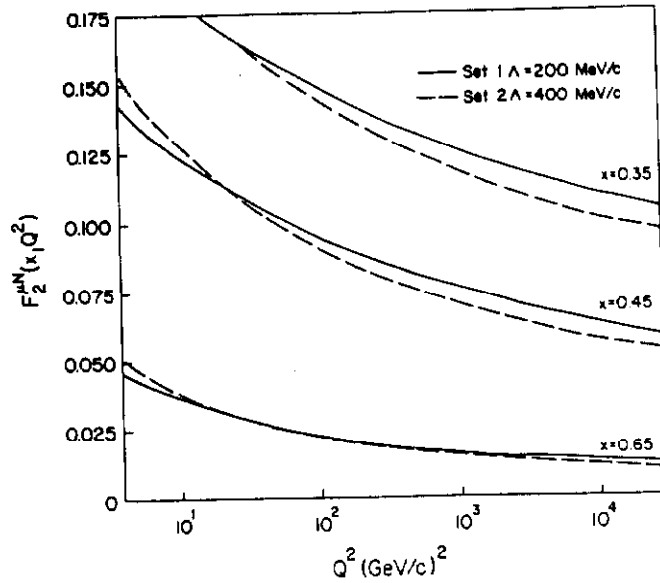


Fig. 3. Predictions for  $F_2$  at several values of  $x$  based on the two sets of distributions.<sup>7</sup> The vertical lines indicate in each case the range in  $Q^2$  over which data were fitted.